Debye mass at the QCD transition in the PNJL model

J. Jankowski,[1,*] D. Blaschke,[2,3,†] and O. Kaczmarek[4,‡]

[1]Institute of Physics, Jagiellonian University ul. Lojasiewicza 11 30-348 Kraków Poland
[2]Institute for Theoretical Physics, University of Wrocław, 50-204 Wrocław, Poland
[3]Bogoliubov Laboratory for Theoretical Physics, Joint Institute for Nuclear Research, RU - 141980 Dubna, Russia
[4]Department of Physics, Bielefeld University, D-33501 Bielefeld, Germany

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We consider colour-electric screening as expressed by the quark contribution to the Debye mass calculated in a PNJL model with emphasis on confining and chiral symmetry breaking effects. We observe that the screening mass is entirely determined by the nonperturbative quark distribution function and temperature dependent QCD running coupling. The role of the gluon background (Polyakov loop) is to provide strong suppression of the number of charge carriers below the transition temperature, as an effect of confinement, while the temperature dependent dynamical quark mass contributes additional suppression, as an effect of chiral symmetry breaking. An alternative derivation of this result from a modified kinetic theory is given, which allows for a slight generalization and explicit contact with perturbative QCD. This gives the possibility to gain insights into the colour screening mechanism in the region near the QCD pseudocritical temperature and to provide a guideline for the interpretation of lattice QCD data.

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I. INTRODUCTION

Theoretical and experimental investigations of quantum chromodynamics (QCD) at finite temperatures are performed with the aim to gain insights to the mechanisms of chiral symmetry restoration and deconfinement. From this perspective the heavy-quark (HQ) potential is one of the most important probes to be studied. In the vacuum the heavy quark-antiquark system is well described in terms of effective field theories, owing to the energy scale separation [1, 2] and the fact that the HQ potential can be properly defined in terms of large Wilson loops. Then the spectrum of charmonium and bottomonium states as solutions of the Schrödinger equation for heavy quarkonia can be extracted and faced experiment. Finite temperature studies rely on the extraction of a static potential from *ab initio* simulations of lattice QCD (lQCD) considering the singlet free energy due to a pair of static color charges as a function of their distance by means of Polyakov loop correlators [3]. However, the systematic field theoretic description within the NRQCD framework [2] is much more subtle and delicate because new variable energy scales like the temperature T or the screening mass m_D enter the problem distorting the energy hierarchy. In addition, from the definition of a real-time potential [4] it was realized that also an imaginary part in the potential appears at finite temperature [5–7]. Using lattice QCD studies of this complex values static potential it was recently found that the real part is well described by the color singlet free energy of a static quark-antiquark pair [8].

While those results shed light on the important question about the identification of the HQ potential with the colour singlet free energy [9] another important question is that of microphysics insights into the screening mechanism. Based on some first principle motivations [10], Riek and Rapp have proposed an ansatz for the HQ potential [11] in the form of a screened Cornell potential where the Coulomb and the linear parts are subject to two different screening masses m_D and $\tilde{m}_D$, respectively. The fit of the temperature dependence of these parameters provided in Ref. [11] using available lQCD data has revealed some unexpected aspects. First, the coulombic Debye mass m_D has a linear behaviour with very small slope (smaller than expected from pQCD). Second, the screening mass of the confining part, $\tilde{m}_D$, shows a strong suppression for temperatures below T_c and a linear rise for high temperatures (higher than expected from pQCD).

This is somehow different from the standard approach, where the lQCD data for the large distance part of this HQ potential are fitted either to a Debye screened Coulomb ansatz or to a form motivated by a Debye-Hückel theory [12–14]. An ansatz taking into account real and imaginary parts of the HQ potential has been recently considered in [14] in the context of quenched lattice data. Here we use $m_D(T)$ from lQCD data with dynamical fermions obtained from fitting the heavy quark-antiquark free energy at large separations to the standard Debye screened potential [12] to compare with the model predictions. While its behaviour well above the pseudocritical temperature T_c of the QCD phase transition can qualitatively be understood in terms of perturbation theory, the interpretation of the lattice data in the vicinity of T_c require essentially nonperturbative approaches addressing effects of confinement and chiral symmetry breaking. The leading order perturbative result reads

*Electronic address: jakubj@th.if.uj.edu.pl
†Electronic address: blaschke@ift.uni.wroc.pl
‡Electronic address: okacz@physik.uni-bielefeld.de

$m_D = (1 + \frac{N_f}{6})^{\frac{1}{2}} gT$, while the next-to-leading order can be obtained by resummation of the leading contribution of the high temperature expansion [15–17]. A detailed understanding of the physics behind the Debye mass in the nonperturbative domain is subject to many current studies.

Approaches based on the operator product expansion (OPE) [18], gauge/gravity duality [19, 20] or phenomenological models [10] make an attempt to give a microscopic description of the screening phenomenon. However, the very definition and numerical determination of a screening mass is obscured by the complications of the nonabelian nature of QCD and the large value of the coupling constant near the QCD transition region [21, 22].

In this paper we investigate screening effects in a PNJL model which proved successful in reproducing various aspects of hadronic excitations in the medium [23] and of lQCD thermodynamics [24]. We will evaluate the one-loop polarization function using PNJL propagators with QED like vertices thus extending a previous calculation made for massless quarks [25]. In this rough way we implement confinement and chiral symmetry breaking effects which in turn allows a comparison to the lattice data for the Debye mass. We will show that one can reproduce the correct shape of its temperature dependence. However, due to the absence of dynamical gluons in our PNJL model calculation, we lack dynamical degrees of freedom and therefore stay below the lQCD result.

It is well known that the screening mass in QED or pQCD can also be derived from kinetic theory [26, 27]. Interrelations between plasma physics and quark gluon plasma are known to bring many relevant insights and physical motivation behind the field theory calculations [28]. We also explore this possibility and modify the standard kinetic theory approach by replacing usual Fermi-Dirac distribution functions for quarks with those modified by the coupling to the Polyakov loop. In this way we are able to reproduce the result for the Debye mass calculated within our model and furthermore to achieve contact with QCD by inclusion of effects of perturbative non-Abelian vertices.

The paper is organized as follows. In section II we outline the model calculation of the Debye mass within the PNJL model, whereby details are referred to the Appendix. Section III presents the kinetic theory approach to the problem where we give a simple and intuitive derivation of the screening mass suggesting a straightforward modification of the standard approach. Section IV is devoted to a comparison with lQCD data and their interpretation while section V gives the conclusions.

II. PNJL MODEL CALCULATION OF THE DEBYE MASS

In reference [25] a model was considered where the vacuum HQ potential was screened by a quark loop with internal lines coupled to a temporal background gluon field. For the static interaction potential $V(q)$, $q^2 = |\mathbf{q}|^2$, the statically screened potential is given by a resummation of one-particle irreducible diagrams (RPA "bubble" resummation)

$$V_{\rm sc}(q) = V(q)/[1 + F(0;\mathbf{q})/q^2] \ , \tag{1}$$

where the longitudinal polarization function in the finite temperature case is defined via the projector decomposition of the self energy (in Euclidean space)

$$\Pi_{\mu\nu}(i\omega, q) = F(i\omega, q) P^L_{\mu\nu} + G(i\omega, q) P^T_{\mu\nu} \ , \tag{2}$$

where the projectors satisfy

$$(P^L)^2 = (P^T)^2 = 1 \qquad P^T P^L = P^L P^T = 0 \ , \tag{3}$$

$$P^L_{\mu\nu} + P^T_{\mu\nu} = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2} \ , \tag{4}$$

$$P^T_{ij} = \delta_{ij} - \frac{Q_i Q_j}{Q^2} \qquad P^T_{44} = P^T_{j4} = P^T_{4j} = 0 \ . \tag{5}$$

Here, $Q = (\omega, \mathbf{q})$ is the Euclidean four momentum. From this we get the gauge invariant longitudinal component

$$F(i\omega, q) = \frac{Q^2}{q^2} \Pi_{44}(i\omega, q) \ , \tag{6}$$

and it can be calculated within thermal field theory as

$$\Pi_{44}(i\omega_l;\mathbf{q}) = g^2 T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \ \mathrm{Tr}[\gamma_4 S_\Phi(i\omega_n;\mathbf{p}) \gamma_4 S_\Phi(i\omega_n - i\omega_l; \mathbf{p} - \mathbf{q})] \ . \tag{7}$$

Here $\omega_l = 2\pi l T$ are the bosonic and $\omega_n = (2n+1)\pi T$ the fermionic Matsubara frequencies of the imaginary-time formalism. The symbol Tr stands for traces in color, flavor and Dirac spaces. S_Φ is the fermionic quark propagator coupled to the homogeneous static gluon background field φ_3. Its inverse is given by [23, 24]

$$S_\Phi^{-1}(\mathbf{p}; i\omega_n) = \gamma \cdot \mathbf{p} - \gamma_4 \omega_n + \gamma_4 \lambda_3 \varphi_3 + m \ , \tag{8}$$

where $\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}$ and $m = m(T)$ is the dynamically generated temperature dependent mass for light quarks as described, e.g., within the NJL model [31]. The variable φ_3 is related to the Polyakov loop variable defined by [24]

$$\Phi(T) = \frac{1}{3}\mathrm{Tr_c}(\mathrm{e}^{\mathrm{i}\beta\lambda_3\varphi_3}) = \frac{1}{3}(1 + 2\cos(\beta\varphi_3)) \ ,$$

The physics of $\Phi(T)$ is governed by the temperature-dependent Polyakov loop potential $\mathcal{U}(\Phi)$, which is fitted to describe the lattice data for the pressure of the pure glue system [24]. After performing the color-, flavor- and Dirac traces and making the fermionic Matsubara summation we obtain [32] (see Appendix A for the details)

$$\begin{aligned} \Pi_{44}(i\omega, q) \ &= \ g^2 \mathrm{Re} \int_0^\infty \frac{p^2 dp}{\pi^2} \frac{2 f_\Phi(E_p)}{E_p} \\ &\qquad \left\{ 1 + \frac{4E_p i\omega + q^2 + \omega^2 - 4E_p^2}{4pq} \ln \frac{R_+(\omega)}{R_-(\omega)} \right\} , \end{aligned} \tag{9}$$

where

$$R_\pm(\omega) = -\omega^2 - q^2 - 2i\omega E_p \pm 2pq \ , \tag{10}$$

and $\mathrm{Re} f(\omega) := \frac{1}{2}(f(\omega) + f(-\omega))$. Taking the static, long wavelength limit [23, 32, 33] we identify, after continuation $i\omega \to q_0 + i\epsilon$ to the Minkowski space, $F(q_0 = 0, q \to 0) = m_D^2(T)$ and whence

$$m_D^2(T) = \frac{16\alpha_s}{\pi} \int_0^\infty dp \ \left[p^2 + E_p^2\right] \frac{f_\Phi(E_p)}{E_p} \ . \tag{11}$$

Here $m_D(T)$ is the Debye mass, $E_p = \sqrt{p^2 + m^2}$ is the quasiparticle dispersion relation for light quarks with $N_c = 3$ colour and $N_f = 2$ flavour degrees of freedom; $f_\Phi(E)$ is the quark distribution function [23]

$$f_\Phi(E) = 3 \frac{\Phi(1 + 2e^{-\beta E})e^{-\beta E} + e^{-3\beta E}}{1 + 3\Phi(1 + e^{-\beta E})e^{-\beta E} + e^{-3\beta E}} \ . \tag{12}$$

This result is different from the standard QED case only in that the Fermi-Dirac distribution has been replaced by the function (12). In comparison to the free fermion case [6, 33] the coupling to the Polyakov loop variable $\Phi(T)$ gives rise to a modification of the Debye mass, encoded in the modification of usual Fermi-Dirac distribution function. In the limit of deconfinement ($\Phi = 1$), the case of a massive quark gas is obtained, while for confinement ($\Phi = 0$) one finds a considerable suppression. The temperature dependence of the Debye mass is shown in figure 1 and as expected from the very beginning it turns out to be below the free massless case, in the confined and in the transition region (with a pseudocritical temperature $T_c \approx 200$ MeV). For temperatures $T >> T_c$ the free gas behaviour $m_D^2 = 2/3 g^2 T^2$ is reproduced.

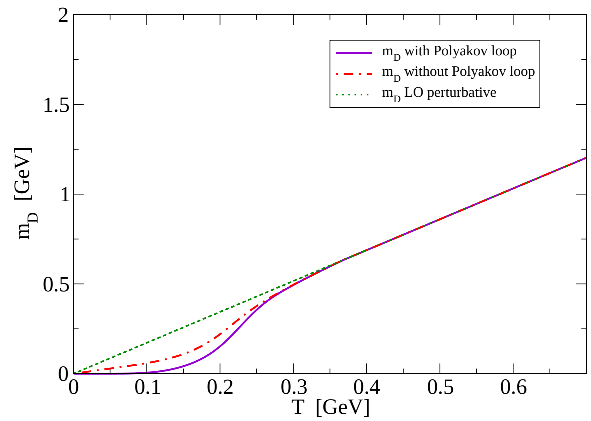

FIG. 1: Debye mass in leading order perturbation theory (dotted line), including the effect of dynamical chiral symmetry breaking without coupling to the Polyakov loop (dash-dotted line) and including the coupling to the Polyakov loop (solid line). Calculated with $\alpha_s = 0.471$ fitted to charmonium spectrum at $T = 0$.

The above result can be also obtained in a different way. It is well known that in QED the Debye mass is related to the pressure via the second derivative [32, 33]

$$m_D^2(T, \mu_e) = e^2 \frac{\partial^2 P(T, \mu_e)}{\partial \mu_e^2} \ , \tag{13}$$

where here μ_e is related to the electric charge of the system. This relation is a consequence of the Dyson-Schwinger equation for the photon self energy and the Ward identity relating the electron-photon vertex to the quark propagator. This is only true in abelian gauge theory and breaks down for non-abelian theories like QCD. On the technical level the proper Ward identity (called then Slavnov-Taylor identity) becomes much more complex and does not allow a simple derivation. Because our calculation in this section is similar to a one-loop QED calculation it is interesting to see if a similar relation holds also in the PNJL model. We check this in the finite temperature and zero chemical potential case by directly evaluating right hand side of eq. (13). Let us recall the quark contribution to the meanfield pressure reads [23]

$$\begin{aligned} P_q(T, \mu) \ &= \ -2N_f T \int \frac{d^3p}{(2\pi)^3} \\ &\qquad \left\{ \ln\left[1 + 3(\bar\Phi + \Phi X_-)X_- + X_-^3\right] \right. \\ &\qquad \left. + \ln\left[1 + 3(\Phi + \bar\Phi X_+)X_+ + X_+^3\right] \right\} , \end{aligned} \tag{14}$$

where $X_\mp = e^{-\beta(E_p \mp \mu)}$. The vacuum pressure has been subtracted and at finite μ the function Φ is generally different from its complex conjugate $\bar\Phi$. In the small density limit constituent quark mass and expectation value

of traced Polyakov loop are μ independent so the second derivative of the quark pressure simplifies giving after noticing that it can be written as derivative with respect to the quasiparticle energy E_p

$$\begin{aligned} m_D^2(T) &= -g^2 \frac{\partial^2 P}{\partial \mu^2}(T, \mu = 0) \\ &= 12 g^2 N_f \int \frac{d^3p}{(2\pi)^3} \frac{d}{dE_p} \left[\frac{X^3 + \Phi(X + 2X^2)}{1 + 3\Phi(X + X^2) + X^3} \right], \end{aligned} \tag{15}$$

where we have used $X = e^{-\beta E_p}$ and the fact that $\Phi = \bar{\Phi}$ for $\mu = 0$. The quantity under the integral can be identified with the energy derivative of our modified distribution function bringing us to the following formula

$$m_D^2(T) = -\alpha_s \frac{8N_f}{\pi} \int_0^\infty dp p^2 \frac{df_\Phi}{dE_p}(E_p) \ , \tag{16}$$

which after integration by parts (see section III) gives for $N_f = 2$ the result (11).

III. KINETIC THEORY APPROACH

The usual Debye screening mass in QED or perturbative QCD can be derived within a kinetic theory approach [26, 29]. In this section we will modify the standard kinetic theory so that we consistently reproduce our previous Debye mass derivation and generalize it in order to make contact with perturbative calculations for high temperatures. The kinetic theory approach has been widely used in the context of perturbative QCD [27, 30] providing a physical picture behind the hard thermal loop approximation and some insights into transport properties and collective modes of the quark gluon plasma. The appropriate change with respect to the textbook result is that the usual Fermi Dirac distribution function is replaced by the Polyakov loop modified distribution function (12). Then the charge density induced by the electrostatic potential $A_0(x) = V(x)$ can be written

$$\begin{aligned} \rho_{\rm ind}(x) &= 2g \int \frac{d^3p}{(2\pi)^3} \left[f_\Phi(E_p + gV(x)) - f_\Phi(E_p - gV(x)) \right] \\ &\approx 4g^2 \int \frac{d^3p}{(2\pi)^3} \frac{df_\Phi}{dE_p}(E_p) V(x) \ , \end{aligned} \tag{17}$$

where the factor 2 is due to the fermion spin. The Maxwell equations give

$$\begin{aligned} -\nabla^2 V(x) &= \rho_{\rm ind}(x) = \frac{2g^2}{\pi^2} \int_0^\infty dp p^2 \frac{df_\Phi}{dE_p}(E_p) V(x) \\ &= -m_D^2(T) V(x) \ , \end{aligned} \tag{18}$$

where

$$\begin{aligned} m_D^2(T) &= -\frac{2N_f g^2}{\pi^2} \int_0^\infty dp p^2 \frac{df_\Phi}{dE_p}(E_p) \\ &= -\frac{8N_f \alpha_s}{\pi} \int_0^\infty dp\, p \sqrt{p^2 + m^2} \frac{df_\Phi(E_p)}{dp} \end{aligned} \tag{19}$$

and the relation $E_p dE_p = p dp$ has been used. Integrating by parts,

$$\begin{aligned} m_D^2(T) &= -\frac{8N_f \alpha_s}{\pi} \int_0^\infty dp\, p \sqrt{p^2 + m^2} \frac{df_\Phi(E_p)}{dp} \\ &= -\frac{8N_f \alpha_s}{\pi} \Big[p \sqrt{p^2 + m^2} f_\Phi(E_p) \big|_{p=0}^{p=\infty} \\ &\quad - \int_0^\infty dp f_\Phi(E_p) \frac{d}{dp} \left(p \sqrt{p^2 + m^2} \right) \Big] \ , \end{aligned} \tag{20}$$

one arrives at the result for the Debye screening mass for N_f flavours of fermions

$$m_D^2(T) = \frac{8N_f \alpha_s}{\pi} \int_0^\infty dp \ \left[p^2 + E_p^2 \right] \frac{f_\Phi(E_p)}{E_p} \ . \tag{21}$$

For the non-abelian case the induced density reads

$$\rho_{\rm ind}(x) = 2g \int \frac{d^3p}{(2\pi)^3} \left[f_+^b(p,x) - f_-^b(p,x) \right] {\rm Tr}[t^b t^a] \ , \tag{22}$$

where ${\rm Tr}[t^b t^a] = \frac{1}{2}\delta^{ab}$ and we assume

$$f_\pm^a(x,p) = \pm g \frac{df_\Phi}{dE_p}(E_p) V^a(x) \ . \tag{23}$$

Doing the same steps as before will give the Debye mass ($N_f = 2$)

$$m_D^{*2}(T) = \frac{8\alpha_s}{\pi} \int_0^\infty dp \ \left[p^2 + E_p^2 \right] \frac{f_\Phi(E_p)}{E_p} \ , \tag{24}$$

which now reproduces the Debye mass of perturbative QCD for high temperatures.

IV. COMPARISON WITH LATTICE QCD

Within lQCD, the temperature dependent screening masses have been defined from the exponential fall-off of the colour singlet free energies [3] and the results could be represented in the rescaled leading order perturbative result

$$\frac{m_D(T)}{T} = A(T) \left(1 + \frac{N_f}{6} \right)^{1/2} g_{2-{\rm loop}}(T) \ , \tag{25}$$

where the factor $A(T)$ was introduced to account for non-perturbative effects. The analysis performed in [12] has shown that $A(T) \approx 1.66$ for $N_f = 2+1$ and $T \geq 1.2\ T_c$. Obviously, $A(T) \to 1$ for high temperatures in agreement with perturbation theory. In order to compare our model with lQCD data we have to adopt equation (24), because it makes contact with perturbation theory for high temperatures. The running coupling constant is modelled in two forms. The first one being the two-loop result [34]

$$\alpha(T) = \frac{1}{4\pi \left[2\beta_0 \ln(\frac{\mu T}{\Lambda}) + (\frac{\beta_1}{\beta_0}) \ln(2 \ln(\frac{\mu T}{\Lambda})) \right]} \ , \tag{26}$$

where $\mu = \pi$ is the upper bound for the perturbative coupling. For Λ we use a value in the $\overline{\mathrm{MS}}$ scheme of $\Lambda_{\overline{MS}} = 260$ MeV and

$$\beta_0 = \frac{1}{16\pi^2}\left(11 - \frac{2N_f}{3}\right) \ , \tag{27}$$

$$\beta_1 = \frac{1}{(16\pi^2)^2}\left(102 - \frac{38N_f}{3}\right) \ . \tag{28}$$

The other possibility is to use a running coupling constant obtained by solving the one-loop renormalization group equation with pole subtraction [35]

$$\alpha_s(T) = \frac{4\pi}{\widetilde{\beta}_0}\left[\frac{1}{2\ln\frac{\pi T}{\Lambda}} + \frac{\Lambda^2}{\Lambda^2 - (\pi T)^2}\right] \ , \tag{29}$$

where $\widetilde{\beta}_0 = 11 - 2/3N_f$.

In this way we can identify our model predictions for the nonperturbative effects of confinement and chiral symmetry restoration which are expressed as a temperature dependent factor $A(T)$ which reads (for $N_f = 2$)

$$A^2(T) = \frac{6}{\pi T^2}\int_0^\infty dp \frac{f_\Phi(E_p)}{E_p}\left\{p^2 + E_p^2\right\} \ . \tag{30}$$

We see that it is entirely controlled by the quark distribution function and by the temperature dependent quark mass which mimic confinement as well as chiral symmetry breaking aspects of strong interaction dynamics.

For a comparison to lattice QCD data we have chosen the $N_t = 6$ data for 2+1-flavors from [12]. There two definitions of the running coupling were used, α_s from a fit using a Debye screened Coloumb Ansatz at large separations and α_{max} obtained at the maximum of the effective r and T-dependent running coupling. Those results are based on an analysis of gauge field configurations generated by the RBC-Bielefeld collaboration in (2+1)-flavor QCD for the calculation of the QCD equation of state [36] where the pion mass is about 220 MeV and the strange quark mass is adjusted to its physical value.

From the lower panel of Fig. 2 we see that the general trend of the lattice data is reproduced. The fact that our result for the Debye mass stays below the lattice data is due to the lack of dynamical gluons which in full QCD also bring a contribution to the screening effects. Also the smoother behaviour around T_c is due to the used form of the running coupling which in principle is not applicable in the transition region. Below T_c the suppression of colour charges (which is meant as a rough form of confinement) drives $m_D(T)$ fastly to zero and overcompensates the increasing interaction strength which taken alone would tend to increase the screening mass. Note that the quenched results for m_D in [12] and [14] lie below the lattice data shown in Fig. 2 due to the missing degrees of freedom dynamical quarks.

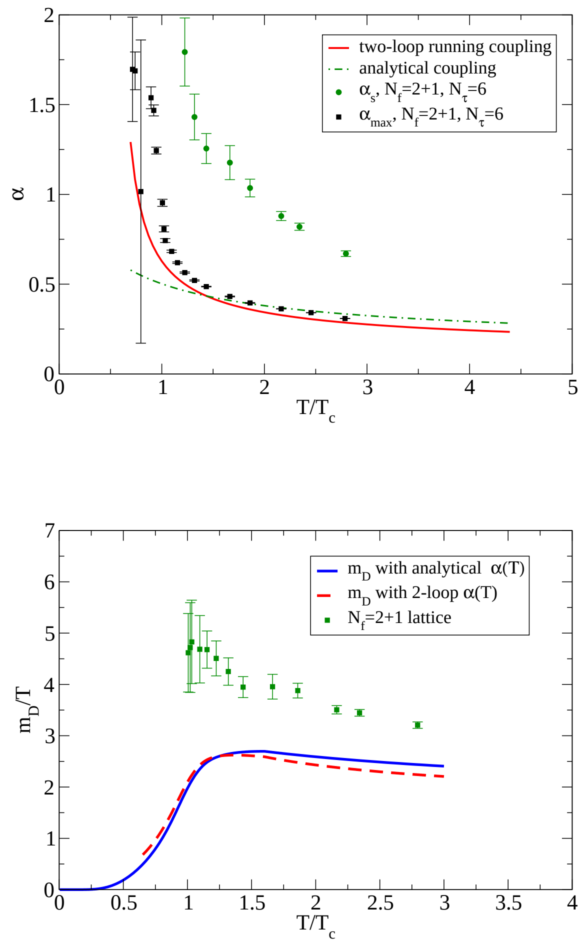

FIG. 2: Upper panel: Two loop running coupling an the analytical running coupling [34, 35]. Lower panel: Debye mass with coupling to the Polyakov loop and running coupling constant $\alpha(T)$ compared with the lattice data [3].

V. CONCLUSIONS

In this short note we have presented a calculation of the quark contribution to the Debye screening mass using the PNJL model which captures chiral dynamics and in a very rough way some aspects of confinement. We have compared our results to the Debye masses extracted from lattice QCD simulations of the static heavy-quark potential and obtain overall agreement for the shape of the temperature dependence. Naturally, our results for the Debye mass stay below the lattice results since in our model the gluon contribution is neglected. However, as the observed gluon contribution on the lattice is of the same shape [12] we are convinced that our model captures the essentials of the influence of chiral dynamics and confinement on the screening of the heavy-quark potential. A further improvement of the calculation would be to include dynamical gluons into the system, to im-

prove the modelling of quark (and gluon) confinement and to elaborate on the behaviour of the running coupling constant. The latter should also be compared with the lattice QCD result for this quantity as it is obtained simultaneously with that for the Debye mass. At this point it is interesting to go back to the different interpretations of the same lattice data. Here we would like to mention the one by Ref. [11] where the ansatz of a screened Corrnell type potential was adopted with two different Debye masses, one for the linear confining part ($\widetilde{m}_D$) and one for the Coulombic part (m_D). The performed fit gave a drastically different behaviour of the two screening masses. The temperature dependence obtained for $\widetilde{m}_D$ appears similar to that of the Debye mass in the present approach, calculated for a T- independent coupling constant (see Fig. 1). The physical reason for such a distinction could be that the stringy and the Coulombic parts of the potential act on different length scales so that the screening of them involves different dynamics. The linear part should be dominant for larger distances thus involving stronger interactions and more correlations in the screening mechanism. Thus one could expect that $\widetilde{m}_D > m_D$ for all temperatures which is the finding of [11]. This is, however, only a qualitative argument. Our calculation, since it is at one-loop order with a QED like interaction should apply to the screened Coulomb potential part from which the lattice QCD result has been extracted. As has been demonstrated here, this comparison provides a reasonable interpretation of the temperature dependence of the Debye mass.

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Appendix A: Polarization loop, temporal component

The calculation we have performed follows in all steps the standard QED evaluation of the polarization loop [32, 33] with the only difference that the usual Matsubara summation is now equipped with a trace over the colour indices [23]. To start with let us define the 44 component of the polarization tensor

$$\Pi_{44} = g^2 T \sum_n \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\left\{ [\gamma_4(m+\gamma_4\omega_n - \vec{\gamma}\vec{p})\gamma_4(m+\gamma_4\omega_n - \gamma_4\omega_l - \vec{\gamma}(\vec{p}-\vec{q}))]\,\Delta(i\omega_n,\vec{p})\Delta(i\omega_n - i\omega_l, \vec{p}-\vec{q}) \right\}, \quad \text{(A1)}$$

where Tr stands for the trace in Dirac and color spaces, $\omega_n = (2n+1)\pi T - A_4$, with the temporal gluon field A_4, $\omega_l = 2\pi l T$. Let us define

$$\Delta(i\omega_n,\vec{p}) = \frac{1}{\omega_n^2 + p^2 + m^2} = \sum_{s=\pm} \frac{s}{2E_p} \frac{1}{i\omega_n + sE_p} \;, \quad \text{(A2)}$$

where $E_p = \sqrt{p^2+m^2}$. The first step is to calculate the Dirac trace using $\{\gamma_\mu,\gamma_\nu\} = -2\delta_{\mu\nu}$

$$\mathrm{Tr}\left[(m+\gamma_4\omega_n + \vec{\gamma}\vec{p})\gamma_4^2(m+\gamma_4\omega_n - \gamma_4\omega_l - \vec{\gamma}(\vec{p}-\vec{q}))\right] = -4\left[m^2 - \omega_n(\omega_n-\omega_l) + \vec{p}(\vec{p}-\vec{q})\right] \;, \quad \text{(A3)}$$

$$\mathcal{N}_V = \omega_n(\omega_n-\omega_l) - \vec{p}(\vec{p}-\vec{q}) - m^2 = \omega_n^2 - \omega_n\omega_l - p^2 + pq - m^2 \;, \quad \text{(A4)}$$

$$\mathrm{Tr}\left[(m+\gamma_4\omega_n + \vec{\gamma}\vec{p})\gamma_4^2(m+\gamma_4\omega_n - \gamma_4\omega_l - \vec{\gamma}(\vec{p}-\vec{q}))\right] = 4\mathcal{N}_V \;. \quad \text{(A5)}$$

$$\Pi_{44}(i\omega_l;\mathbf{q}) = \frac{g^2T}{2} T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\left\{ 2\Delta(i\omega_n,\vec{p}) + (4pq - 4E_p^2 - q^2 - \omega_l^2)\Delta(i\omega_n,\vec{p})\Delta(i\omega_n - \omega_l, \vec{p}-\vec{q}) \right\} . \quad \text{(A6)}$$

Now we want to decompose (A34) so that we have $\Delta(i\omega_n,\vec{p}) + \Delta(i\omega_n - i\omega_l, \vec{p}-\vec{q})$ and for that we introduce

$$\mathcal{M} = p^2 + 2m^2 + \omega_n^2 + (\omega_n-\omega_l)^2 + (p-q)^2 = 2(p^2 + m^2 + \omega_n^2 - \omega_n\omega_l - pq) + \omega_l^2 + q^2 \;, \quad \text{(A7)}$$

$$2\mathcal{N}_V = \mathcal{M} + 4pq - 4p^2 - 4m^2 - q^2 - \omega_l^2 \;. \quad \text{(A8)}$$

Further, we evaluate three coloured Matsubara sums

$$J_3 = T\sum_n \mathrm{Tr}_c\left[\Delta(i\omega_n,\vec{p})\Delta(i\omega_n - i\omega_l, \vec{p}-\vec{q})\right] \;, \quad \text{(A9)}$$

$$J_1 = T\sum_n \mathrm{Tr}_c \Delta(i\omega_n,\vec{p}) \;. \quad \text{(A10)}$$

$$J_2 = T\sum_n \mathrm{Tr}_c\left\{[\vec{p}(\vec{p}-\vec{q})]\Delta(i\omega_n,\vec{p})\Delta(i\omega_n - i\omega_l, \vec{p}-\vec{q})\right\} \;, \quad \text{(A11)}$$

following [23] by going to the Polyakov gauge (where Polyakov loop variable is diagonal) and explicitly calculating the trace. The outcome reads

$$J_3 = \sum_{s,s'=\pm} \frac{ss'}{4E_pE_{p-q}} \frac{1}{i\omega + sE_p - s'E_{p-q}} \left[f_\Phi(sE_p) - f_\Phi(s'E_{p-q})\right] \;, \quad \text{(A12)}$$

$$J_3 = \frac{1}{4E_pE_{p-q}} \left\{ [f_\Phi(E_p) - f_\Phi(E_{p-q})] \left(\frac{1}{i\omega + E_p - E_{p-q}} - \frac{1}{i\omega - E_p + E_{p-q}} \right) + \right. \quad \text{(A13)}$$
$$\left. [1 - f_\Phi(E_p) - f_\Phi(E_{p-q})] \left(\frac{1}{i\omega + E_p + E_{p-q}} - \frac{1}{i\omega - E_p - E_{p-q}} \right) \right\} ,$$

$$J_2 = \sum_{s,s'=\pm} \frac{ss'}{4E_pE_{p-q}} \frac{\vec{p}(\vec{p}-\vec{q})}{i\omega + sE_p - s'E_{p-q}} \left[f_\Phi(sE_p) - f_\Phi(s'E_{p-q})\right] \ , \tag{A14}$$

$$J_2 = \frac{\vec{p}(\vec{p}-\vec{q})}{4E_pE_{p-q}} \Bigg\{ \left[f_\Phi(E_p) - f_\Phi(E_{p-q})\right] \left(\frac{1}{i\omega + E_p - E_{p-q}} - \frac{1}{i\omega - E_p + E_{p-q}} \right) + \\ \left[1 - f_\Phi(E_p) - f_\Phi(E_{p-q})\right] \left(\frac{1}{i\omega + E_p + E_{p-q}} - \frac{1}{i\omega - E_p - E_{p-q}} \right) \Bigg\} \ , \tag{A15}$$

$$J_4 = \sum_{s=\pm} \frac{s}{2E_p} f_\Phi(-sE_p) = \frac{1}{2E_p}\left[1 - 2f_\Phi(E_p)\right] \ , \tag{A16}$$

which is in agreement with (6.27) and (6.37) from LeBellac, with the only difference that Fermi-Dirac distribution functions have been replaced with the modified ones [23]

$$f_\Phi(E) = T\sum_n \mathrm{Tr}_c \left[\frac{1}{i\omega_n - E}\right] = 3\frac{\Phi(1+2e^{-\beta E})e^{-\beta E} + e^{-3\beta E}}{1 + 3\Phi(1+e^{-\beta E})e^{-\beta E} + e^{-3\beta E}} \ . \tag{A17}$$

To obtain the last equation we use the fact that in this specific gauge the Polyakov loop variable is diagonal and that after a Matsubara summation we get

$$f_\Phi(E) = \sum_{j=1}^{3} \frac{1}{1 + e^{\beta A_{jj}} e^{\beta E}} = -\frac{1}{\beta}\frac{\partial}{\partial E} \sum_{j=1}^{3} \ln(1 + L_{jj}e^{-\beta E}) \ , \tag{A18}$$

where $L_{jj} = e^{-\beta A_{jj}}$ and A is to be understood as a temporal component of the gauge field. The evaluation of the colour trace is now trivial and results in

$$\sum_{j=1}^{3} \ln(1 + L_{jj}e^{-\beta E}) = \ln\left[(1 + L_{11}e^{-\beta E})(1 + L_{22}e^{-\beta E})(1 + L_{33}e^{-\beta E})\right] \ . \tag{A19}$$

Using $L_{11} + L_{22} + L_{33} = \mathrm{Tr}L = \mathrm{Tr}L^\dagger = 3\Phi$, $L_{11}L_{22}L_{33} = \det L = \det L^\dagger = 1$ we get

$$f_\Phi(E) = -\frac{1}{\beta}\frac{\partial}{\partial E} \ln[1 + 3\Phi(1 + e^{-\beta E})e^{-\beta E} + e^{-3\beta E}] \ , \tag{A20}$$

which is the modified distribution function (12). We also show that

$$f_\Phi(x + i\omega_l) = f_\Phi(x) \ , \qquad\qquad f_\Phi(-E) = 1 - f_\Phi(E) \ . \tag{A21}$$

Now to get rid of the $f(E_{p-q})$ terms whenever we meet them we change the variables according to [31] $p \to -p + q$ so that $E_{p-q} \to E_p$, $E_p \to E_{p-q}$ and $\vec{p}(\vec{p}-\vec{q})$ does not change. We also make the Wick rotation $i\omega \to \omega$

$$J_3 = \frac{f_\Phi(E_p)}{E_p} \left[\frac{1}{(\omega + E_p)^2 - E_{p-q}^2} + \frac{1}{(\omega - E_p)^2 - E_{p-q}^2} \right] \ , \tag{A22}$$

$$J_2 = \vec{p}(\vec{p}-\vec{q}) \frac{f_\Phi(E_p)}{E_p} \left[\frac{1}{(\omega + E_p)^2 - E_{p-q}^2} + \frac{1}{(\omega - E_p)^2 - E_{p-q}^2} \right] = \vec{p}(\vec{p}-\vec{q}) J_3 \ . \tag{A23}$$

$$E_p^2 - E_{p-q}^2 = 2pq - q^2 \qquad \vec{p}\vec{q} = pq\lambda \qquad \int \frac{d^3p}{(2\pi)^3} = \int \frac{dpp^2d\lambda}{(2\pi)^2} \ . \tag{A24}$$

$$(\omega \pm E_p)^2 - E_{p-q}^2 = \omega^2 - q^2 \pm 2\omega E_p + 2pq\lambda \ , \tag{A25}$$

$$2pq\lambda \to \lambda \; . \tag{A26}$$

We are now left with two angular integrals

$$\int_{-2pq}^{2pq} \frac{d\lambda}{\omega^2 - q^2 \pm 2\omega E_p + \lambda} = \ln \frac{\omega^2 - q^2 \pm 2\omega E_p + 2pq}{\omega^2 - q^2 \pm 2\omega E_p - 2pq} \; , \tag{A27}$$

$$\int_{-2pq}^{2pq} \frac{\lambda d\lambda}{\omega^2 - q^2 \pm 2\omega E_p + \lambda} = 4pq - [\omega^2 - q^2 \pm 2\omega E_p] \ln \frac{\omega^2 - q^2 \pm 2\omega E_p + 2pq}{\omega^2 - q^2 \pm 2\omega E_p - 2pq} \; , \tag{A28}$$

$$R_\pm(\omega) = \omega^2 - q^2 - 2\omega E_p \pm 2pq \; . \tag{A29}$$

Using the definition $\mathrm{Re} f(\omega) = \frac{1}{2}[f(\omega) + f(-\omega)]$ we obtain for the longitudinal component of (2) the result

$$F(\omega, q) = -g^2 \frac{\omega^2 - q^2}{q^2} \mathrm{Re} \int_0^\infty \frac{p^2 dp}{\pi^2} \frac{2 f_\Phi(E_p)}{E_p} \left\{ 1 + \frac{4E_p\omega + q^2 - \omega^2 - 4E_p^2}{4pq} \ln \frac{R_+(\omega)}{R_-(\omega)} \right\} . \tag{A30}$$

To get the electric screening mass (Debye mass) we have to compute $F(0, q \to 0) = m_D^2$, i.e.,

$$F(0, q) = g^2 \int_0^\infty \frac{p^2 dp}{\pi^2} \frac{f_\Phi(E_p)}{E_p} \left\{ 2 + \frac{q^2 - 4E_p^2}{4pq} \ln \frac{(2p - q)^2}{(2p + q)^2} \right\} , \tag{A31}$$

$$\lim_{q \to 0} \frac{1}{q} \ln \frac{(2p - q)^2}{(2p + q)^2} = -\frac{2}{p} \; , \tag{A32}$$

$$F(0, q \to 0) = m_D^2 = g^2 \int_0^\infty \frac{dp}{\pi^2} \frac{2 f_\Phi(E_p)}{E_p} \left\{ p^2 + E_p^2 \right\} \; . \tag{A33}$$

This result has a structure exactly as the QED Debye mass with the only difference that the Fermi-Dirac distribution function is replaced with the Polyakov loop suppressed distribution. After including the factors $N_f = 2$ and $\alpha_s = g^2/4\pi$ we get

$$m_D^2 = \frac{16\alpha_s}{\pi} \int_0^\infty dp \frac{f_\Phi(E_p)}{E_p} \left\{ p^2 + E_p^2 \right\} \; , \tag{A34}$$

which is the formula (11) from the text.